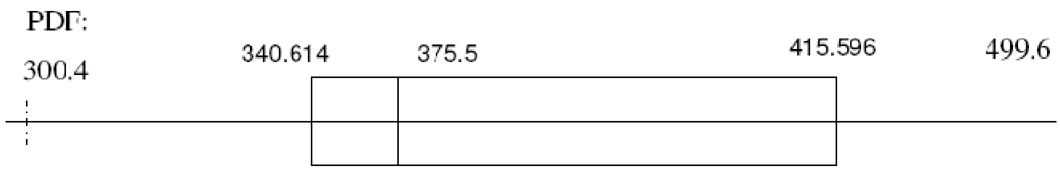
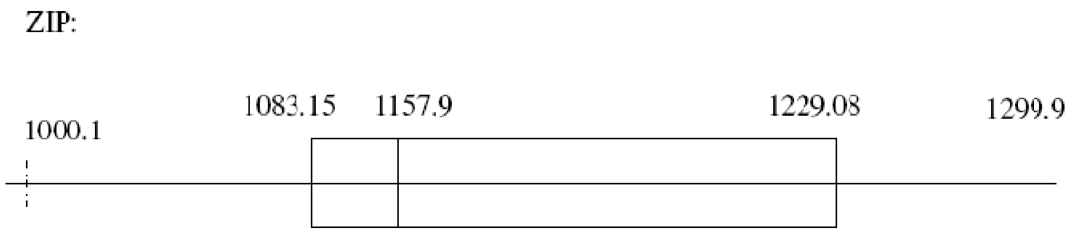
Question 6-3

| | min | 25th Percentile | 50th percentile | 75th percentile | max |
|---|---|---|---|---|---|
| PDF | 300.4 | 340.6144409 | 375.5 | 415.5960999 | 449.6 |
| ZIP | 1000.1 | 1083.148682 | 1157.9 | 1229.081299 | 1299.9 |

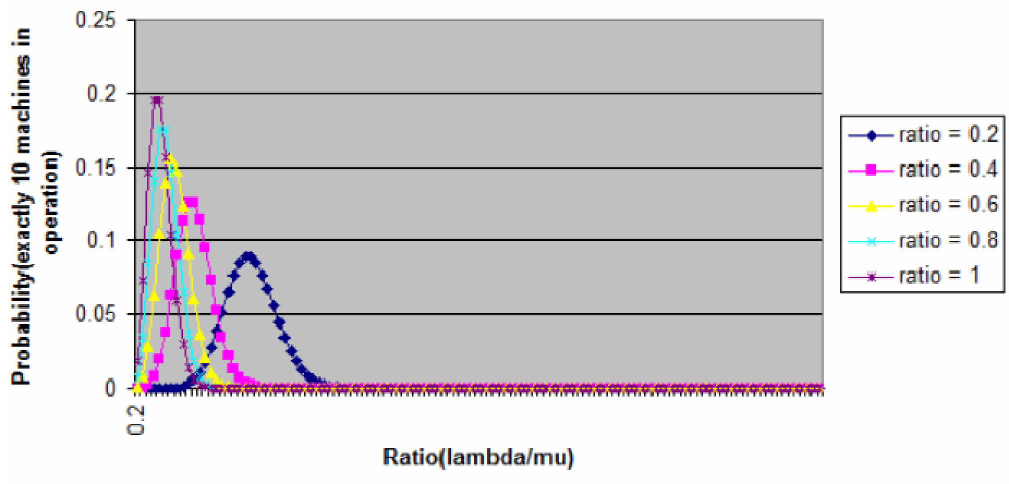


By varying lambda, different ratios of lambda/mu were calculated. These were plugged into the excel spreadsheet and the probabilities were obtained.

These probabilities are for exactly m (here 10) machines being in operation.

A plot was then constructed with the probabilities plotted against the ratio. The following chart was obtained.



Question 7-4

| M | Average time to repair a machine (in minutes) | Repair Rate (mu) | Throughput | N0 | Nf | MTTR | % Machines in Operation | lambda/mu ratio |
|---|---|---|---|---|---|---|---|---|
| 120 | 10 | 0.100 | 0.235 | 117.56 | 2.4 | 10.4 | 98.0% | 0.02 |
| 120 | 12 | 0.083 | 0.234 | 116.97 | 3.0 | 12.9 | 97.5% | 0.024 |
| 120 | 15 | 0.067 | 0.232 | 115.80 | 4.2 | 18.1 | 96.5% | 0.03 |
| 120 | 18 | 0.056 | 0.228 | 113.81 | 6.2 | 27.2 | 94.8% | 0.036 |
| 120 | 20 | 0.050 | 0.223 | 111.49 | 8.5 | 38.1 | 92.9% | 0.04 |
| 120 | 25 | 0.040 | 0.198 | 99.09 | 20.9 | 105.5 | 82.6% | 0.05 |
| 120 | 30 | 0.033333 | 0.167 | 83.33 | 36.7 | 220.1 | 69.4% | 0.06 |
| 120 | 35 | 0.028571 | 0.143 | 71.43 | 48.6 | 340.0 | 59.5% | 0.07 |
| 120 | 40 | 0.025 | 0.125 | 62.50 | 57.5 | 460.0 | 52.1% | 0.08 |
| 120 | 45 | 0.022222 | 0.111 | 55.56 | 64.4 | 580.0 | 46.3% | 0.09 |
| 120 | 50 | 0.02 | 0.100 | 50.00 | 70.0 | 700.0 | 41.7% | 0.1 |
| 120 | 55 | 0.018182 | 0.091 | 45.45 | 74.5 | 820.0 | 37.9% | 0.11 |
| 120 | 60 | 0.016667 | 0.083 | 41.67 | 78.3 | 940.0 | 34.7% | 0.12 |
| 120 | 65 | 0.015385 | 0.077 | 38.46 | 81.5 | 1060.0 | 32.1% | 0.13 |
| 120 | 70 | 0.014286 | 0.071 | 35.71 | 84.3 | 1180.0 | 29.8% | 0.14 |
| 120 | 75 | 0.013333 | 0.067 | 33.33 | 86.7 | 1300.0 | 27.8% | 0.15 |
| 120 | 80 | 0.0125 | 0.062 | 31.25 | 88.8 | 1420.0 | 26.0% | 0.16 |
| 120 | 85 | 0.011765 | 0.059 | 29.41 | 90.6 | 1540.0 | 24.5% | 0.17 |
| 120 | 90 | 0.011111 | 0.056 | 27.78 | 92.2 | 1660.0 | 23.1% | 0.18 |
| 120 | 95 | 0.010526 | 0.053 | 26.32 | 93.7 | 1780.0 | 21.9% | 0.19 |
| 120 | 100 | 0.01 | 0.050 | 25.00 | 95.0 | 1900.0 | 20.8% | 0.2 |
| 120 | 105 | 0.009524 | 0.048 | 23.81 | 96.2 | 2020.0 | 19.8% | 0.21 |
| 120 | 110 | 0.009091 | 0.045 | 22.73 | 97.3 | 2140.0 | 18.9% | 0.22 |
| 120 | 115 | 0.008696 | 0.043 | 21.74 | 98.3 | 2260.0 | 18.1% | 0.23 |
| 120 | 120 | 0.008333 | 0.042 | 20.83 | 99.2 | 2380.0 | 17.4% | 0.24 |
| 120 | 125 | 0.008 | 0.040 | 20.00 | 100.0 | 2500.0 | 16.7% | 0.25 |
| 120 | 130 | 0.007692 | 0.038 | 19.23 | 100.8 | 2620.0 | 16.0% | 0.26 |
| 120 | 135 | 0.007407 | 0.037 | 18.52 | 101.5 | 2740.0 | 15.4% | 0.27 |
| 120 | 140 | 0.007143 | 0.036 | 17.86 | 102.1 | 2860.0 | 14.9% | 0.28 |
| 120 | 145 | 0.006897 | 0.034 | 17.24 | 102.8 | 2980.0 | 14.4% | 0.29 |
| 120 | 150 | 0.006667 | 0.033 | 16.67 | 103.3 | 3100.0 | 13.9% | 0.3 |
| 120 | 155 | 0.006452 | 0.032 | 16.13 | 103.9 | 3220.0 | 13.4% | 0.31 |
| 120 | 160 | 0.00625 | 0.031 | 15.63 | 104.4 | 3340.0 | 13.0% | 0.32 |
| 120 | 165 | 0.006061 | 0.030 | 15.15 | 104.8 | 3460.0 | 12.6% | 0.33 |
| 120 | 170 | 0.005882 | 0.029 | 14.71 | 105.3 | 3580.0 | 12.3% | 0.34 |
| 120 | 175 | 0.005714 | 0.029 | 14.29 | 105.7 | 3700.0 | 11.9% | 0.35 |
| 120 | 180 | 0.005556 | 0.028 | 13.89 | 106.1 | 3820.0 | 11.6% | 0.36 |

8.8

Ve= 1
Vh = 1
Va = 0.725
Vs = 1.75
Vb = 0.29

Given that gamma = 10
Lambda(entry) = gamma * Ventry = 10*1 = 10
Lambda(home) = gamma * Vhome = 10*1 = 10
Lambda(add) = gamma * Vadd = 10*0.725 = 7.25
Lambda(search) = gamma * Vsearch = 10*1.75 = 17.5
Lambda(buy) = gamma * Vbuy = 10*0.29 = 2.9

Total utilization = sum (r = 1 to 6) (lambda_r*D_i,r)
For CPU: 10*0 + 10*0.010 + 7.25*0.010 + 17.5*0.015 + 2.9* 0.020 =
0 + 0.1 + 0.0725 + 0.2625 + 0.058 = 0.493
For Disk: 10*0 + 10*0.015 + 7.25*0.015 + 17.5*0.025 + 2.9*0.010 =
0 + 0.15 + 0.10875 + 0.4375 + 0.0029 = 0.69915

Residence time_K = sum(i=1 to r) Di,K/1-UK
=> Residence time at CPU: (0.01+0.015+0.01+0.02)/(1-0.493) = 0.055/0.507 = 0.108 sec
=> Residence time at Disk: (0.015+0.025+0.015+0.01)/(1-0.69915) = 0.055/0.30085 = 0.1828 sec

Response time_r = sum(i=1 to K) D_i,r/(1-U_i)
Response time_Home = 0.01/(1-0.493)+0.015/(1-0.69915) = 0.01/0.507 + 0.015/0.30085 = 0.0197 + 0.04986 = 0.0696 sec
Response time_Search = 0.015/(1-0.493)+0.025/(1-0.69915) = 0.015/0.507 + 0.025/0.30085 = 0.0296 + 0.0831 = 0.1127 sec
Response time_Add = 0.01/(1-0.493)+0.015/(1-0.69915) = 0.01/0.507 + 0.015/0.30085 = 0.0197 + 0.04986 = 0.0696 sec
Response time_Pay = 0.02/(1-0.493)+0.010/(1-0.69915) = 0.02/0.507 + 0.01/0.30085 = 0.03945 + 0.03324 = 0.07269 sec

10-1

Father's question: What percentage of days is neither son drinking in Leeds?

The percentage of one son drinking is 0.2644, so the percentage of not drinking is 1- 0.2644 = 0.7356.
Therefore, the percentage of neither son drinking is 0.7356 * 0.7356 = 0.5411 = 54.11%

Lake District relatives' question

The percentage of one son kayaking is 0.2308, so the percentage of at least one son kayaking is 1 – (1-0.2308) * (1 – 0.2308) = 0.4083.
So it will take 1 / 0.4083 – 1 = 1.4492 days before one son returns.

Policeman's question

The percentage of one son drinking is 0.2644, so the percentage of at least one son drinking is 1 – (1-0.2644) * (1 – 0.2644) = 0.4589.
So in one month (30 days), 30 * 0.4589 = 13.77 days will find at least one son drinking. Because there is only probability of 0.6 for son to go to London after drinking, the policeman will find at least one son on road to London 13.77 * 0.6 = 8.262 days a month.

Kayak renters' question

The percentage of one son kayaking is 0.2308, so the percentage of at least one son drinking is 1 – (1-0.2308) * (1 – 0.2308) = 0.4083.
So in one month (30 days), 30 * 0.4083 = 12.249 days will find at least one son kayaking. During these days, renters must make a kayak available.

10-6

Queue length = 2

Arrival rate  = 6

10-11

Because the probability of k active connections is equal to the probability of (M-K) failed connections, taking into the consideration of result of Chapter 7, we can have:

$$P_k = P'_{(M-k)\text{ failed}} = \begin{cases} P_0 (\frac{\lambda}{\mu})^{M-k} \binom{M}{M-k} & k = 1, \dots, N \\ P_0 (\frac{\lambda}{\mu})^{M-k} \binom{M}{M-k} \frac{N^{N-M+k} * (M-k)!}{N!} & k = (N+1), \dots, M \end{cases}$$

When all of the ports are busy, it is under condition of k = 0, so $P_{busy} = P_0$.

$$P_{busy} = P_0 = P'_{M\text{ failed}} = [\sum_{k=0}^{N} \left(\frac{\lambda}{\mu}\right)^M \binom{M}{M} + \sum_{k=N+1}^{M} (\frac{\lambda}{\mu})^M \binom{M}{M} \frac{N^{N-M} * (M)!}{N!}]^{-1}$$

$$= [\sum_{k=0}^{N} \left(\frac{\lambda}{\mu}\right)^M + \sum_{k=N+1}^{M} (\frac{\lambda}{\mu})^M \frac{N^{N-M} * M!}{N!}]^{-1}$$