Schema Theorem

Probability of selecting individual *i* in one spin is

$$m(i,t) * \frac{f_i}{\sum_{j=1}^{n} f_j}$$

A complete mating pool is generated in *n* spins so the number of wins for *i* is

$$n * m(i,t) * \frac{f_i}{\sum_{j=1}^{n} f_j} = m(i,t) \frac{f_i}{\bar{f}}$$

For a particular schema *H*, we have

$$m(H, t+1) = m(H,t) \frac{f(H)}{\bar{f}}$$

Scheme *H* tends to survive crossover if it's not disrupted. The probability of survival is

$$(1 - p_c (\frac{\delta(H)}{l-1}))$$

Schema H tends to survive mutation with probability

$$(1 - (p_m o(H)))$$

Combining these terms we have the fundamental theorem of genetic algorithms, or the schema theorem. It indicates the expected number of individuals containing schema H in the next generation.

$$m(H,t+1) \geq m(H,t)\frac{f(H)}{\bar{f}}[1-p_c\frac{\delta(H)}{l-1}-p_m*o(H)]$$

The schema theorem indicates exponential growth for consistently above average schemas. For example

$$(((10*1.5)*1.5)*1.5)=10*1.5^3$$

Implicit Parallelism

The number of schemas that survive crossover to be evaluated, selected, and recombined is at least

$$O(n^3)$$